Kowshik Bettadapura - Research Statement

Introduction. Complex and algebraic supergeometry is a relatively new area of research. It has antecedent roots in the discovery of supersymmetry in theoretical physics. Its central objects are supermanifolds which authors such as Manin [Man88] and Varadarajan [Var04] have described as 'mildly non-commuting spaces' [Ber87, Var04]. Supergeometry is often featured as a core subject area in physics and mathematics, with chapters devoted to it in canonical textbooks such as [DM99] and Kapranov's entry in [AC21] (available at [Kap15]).

My research in supergeometry falls into two distinct, but related areas: (1) theoretical foundations and (2) applications. These are briefly summarized below and followed by a section outlining my current and future research.

Foundations of complex supergeometry. In my doctoral thesis [Bet16], I looked at the problem of classifying complex structures on supermanifolds. I showed how one can study complex structures on a supermanifold *as* certain, 'twisted' deformations of its body - a classical complex manifold. These results appeared in the article [Bet19c], culminating in a Kuranishi type classification.

The existence of 'exotic, complex structures' on a smooth supermanifold present difficulties in studying global properties of the moduli space of its complex structures. In [Bet18], I studied exotic structures detail and gave characterizations pertaining to their existence.

In the interests of shifting viewpoint, from objects to morphisms, I generalized the concept of splitting type from supermanifolds to morphisms between supermanifolds in [Bet20a]. To illustrate its utility, I gave a new proof of the classical fact that the superspace quadric 'fails to split' as a superspace.

In the article [Bet20b], I continued my work in [Bet20a] to study weighted projective superspace and its subvarieties. Based on the weighting of the projective superspace in question, I was able to obtain address the question of when subvarieties (locally) of the form $f + \theta^j g = 0$ would 'fail to split'.

The articles [Bet20a, Bet20b] were inspired by a mirror symmetry proposal for superspaces by authors Sethi [Set94] and Aganagic and Vafa in [AV04]. Based on my work, I generalized this to complex superspaces by proposing that, under mirror symmetry, the Kähler parameter gets exchanged with the (primary) obstruction to splitting.

Supermoduli theory. Super Riemann surfaces (SRS) and their moduli of complex structures, referred to as 'supermoduli space', feature prominently in superstring theory. A well known result by Donagi and Witten [DW15, DW14] relates the issue of classifying supermoduli space with that of calculating superstring scattering amplitudes.

In [Bet19d], I present a local study of supermoduli space and propose the conjecture that *there are no exotic structures* on SRS deformations. This conjecture is verified explicitly for (odd) deformations of second order. Its consequence for the global structure of supermoduli space remains to be understood.

My work on SRS deformations is continued in [Bet19a] where I derive, in analogy with classical Kodaira-Spencer theory, a correspondence between Čech theoretic, SRS deformations and perturbations of the (super) Dolbeault operator. This allowed me to express Donagi and Witten's gauge pairing formula for the Dolbeault operator in [DW14] as a pairing formula between Čech cycles (gluing data) of an SRS deformation.

Donagi and Witten's aforementioned gauge pairing formula was used to justify the *failure* for supermoduli space to split in genus $g > 1$ with $n > 0$ markings. The pairing holds for any genus and marking however. In [Bet21], I used it to rigorously recover a classical result by D'Hoker and Phong stating, in genus $g = 2$ with no markings, that supermoduli space will in fact split.

In a joint work with H. Lin, [BL21], we present a review of Deligne's compactification of supermoduli space; degenerations of the superstring measure following Witten in [Wit13] and identify conditions under which contributions to the superstring amplitude from the boundary of Deligne's supermoduli compactification will vanish.

Further research. There remains much to be understood in complex and algebraic supergeometry. Below is a list of research topics I am actively pursuing.

- *Variations of splitting type.* In the preprint [Bet19b], I am primarily motivated by the question of whether a given, complex structure on a supermanifold is 'exotic'. Following classical lines of approach, I look to form families of supermanifolds and characterize how the splitting type of the fibers with vary. Further natural questions, arising in analogy with Hodge theory, are:
  - *does there exist a notion of 'splitting type variation' in analogy with variation of Hodge structures?*
  - *does there exist a Gauss-Manin type connection characterizing the splitting type in analogy with Hodge structures?*
  - *does there exist a notion of stability for splitting types, serving to distinguish the 'exotic structures'?*
- *Supermoduli theory in low genus.* Donagi and Witten's proof that supermoduli space fails to split is applicable in genus $g > 4$, unmarked, and for $g > 1$ with $g + 1 > n > 1$ markings. Little else is known outside of these ranges. It is suspected that supermoduli space fails to split in genus $g = 3$ and $g = 4$, unmarked. My work in [Bet21], in genus $g = 2$ (unmarked), clarifies why $g = 2$ supermoduli will split. The general case of arbitrary genus and marking (g, n) seems intractable at this stage. However, I look to extend my analysis in [Bet21] to $g = 3, 4$ and comment on the case where supermoduli space has been compactified.
- *Super Teichmüller theory.* Penner and Zeitlin in [PZ19] have recently described the analogue of the Weil-Petersson form on the decorated super Teichmüller space of a marked surface. This gives super Teichmüller space the structure of a (smooth), supersymplectic supermanifold. It is an open question as to how one might precisely calculate the volume of the form described by Penner and Zeitlin and whether it recovers the volume of supermoduli space in analogy with the classical case. Penner and Zeitlin's work is a new development in supergeometry and its full ramifications remain to be discovered.

References

[AC21] M. Anel and G. Catren, editors. *New spaces in mathematics and physics*. Cambridge University Press, 2021.

[AV04] M. Aganagic and C. Vafa. Mirror symmetry and supermanifolds. *Adv. Theor. and Math. Phys.*, 8:939–54, 2004.

[Ber87] F. A. Berezin. *Introduction to Superanalysis*. D. Reidel Publishing Company, 1987.

[Bet16] K. Bettadapura. *Obstruction Theory for Supermanifolds and Deformations of Superconformal Structures*. PhD thesis, The Australian National University, hdl.handle.net/1885/110239, December 2016.

[Bet18] K. Bettadapura. Higher obstructions of complex supermanifolds. *SIGMA*, 14(094), 2018.

[Bet19a] K. Bettadapura. Analytic and algebraic deformations of super Riemann surfaces. Available at: arXiv:1911.07118, 2019.

[Bet19b] K. Bettadapura. Families of supermanifolds: Splitting types and obstruction maps. Available at arXiv:1906.02391, 2019.

[Bet19c] K. Bettadapura. Obstructed Thickenings and Supermanifolds. *J. Geom. and Phys.*, 139:25–49, 2019.

[Bet19d] K. Bettadapura. On the problem of splitting deformations of super Riemann surfaces. *Lett. Math. Phys.*, 109(2):381–402, February 2019.

[Bet20a] K. Bettadapura. Embeddings of complex supermanifolds. *Adv. Theor. and Math. Phys.*, 6(23):1427–66, 2020.

[Bet20b] K. Bettadapura. Projective superspace varieties, superspace quadrics and non-splitting. *Doc. Math.*, (23):65–91, 2020.

[Bet21] K. Bettadapura. On the splitting of genus two supermoduli. Available at arXiv:2106.16035, July 2021.

[BL21] K. Bettadapura and H. Lin. Boundary contributions to three loop superstring amplitudes. *Journal of Math. Phys.*, 62(4), 2021.

[DM99] P. Deligne and J. W. Morgan. *Quantum Fields and Strings: A course for Mathematicians*, volume 1, chapter Notes on Supersymmetry (following Joseph Bernstein), pages 41–97. American Mathematical Society, Providence, 1999.

[DW14] R. Donagi and E. Witten. Super Atiyah classes and obstructions to splitting of supermoduli space. *Pure and applied mathematics quarterly*, 9(4), 2014.

[DW15] R. Donagi and E. Witten. Supermoduli space is not projected. In *Proc. Symp. Pure Math.*, volume 90, pages 19–72, 2015.

[Kap15] M. Kapranov. Supergeometry in mathematics and physics. available at: arXiv:1512.07042 [math.AG], 2015.

[Man88] Y. Manin. *Gauge Fields and Complex Geometry*. Springer-Verlag, 1988.

[PZ19] R. Penner and A. Zeitlin. Decorated super Teichmueller space. *J. Diff. Geom.*, 111(3):527–66, March 2019.

[Set94] S Sethi. Supermanifolds, rigid manifolds and mirror symmetry. *Nuc. Phys. B*, 430(1):31–50, 1994.

[Var04] V. S. Varadarajan. *Supersymmetry for Mathematicians: An Introduction*. American Mathematical Society, 2004.

[Wit13] E. Witten. Notes on holomorphic string and superstring theory measures of low genus. available at: arXiv:1306.3621 [hep-th], 2013.

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